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Optimum Design Approach for Filament Wound Composite Toroidal Pressure Vessels

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Abstract - Nowadays, toroidal pressure vessels have found a lot of applications in engineering fields as hydrogen gas, gaseous fuel and compressed natural gas vessels. This is due to that these types of pressure vessels possess high volume-t0-weight ratio and high pressure-to-weight ratio which make them of high storage efficiency tanks. In this paper, a design approach is presented to make an optimum design for those types of pressure vessels. The study has focused on the main parameters affecting the optimum design as the cross-section of the toroid (toroid radius, tube radius and thickness), reinforcing fibrous material and the orientation angle of the fibers around the vessel. In addition, three types of composite materials: E-glass/epoxy, Kevlar-49/epoxy, and carbon/epoxy (Thornel 300) are investigated.The results of study have showed that the critical zone of these vessels is at the internal side of the vessel wall, whereas the safest zone is at the external side along the circumferential direction. Also, the study has showed that the optimum cross-section of the toroid is at toroid-t0-tube radii ratio (R/r)=2.6 or more which minimizes the von-Mises stress ratio across the vessel wall and so reduce or eliminate the stress non-uniformity around the vessel wall. Hence, the stress non-uniformity around the vessel wall is reduced or eliminated. The results have also showed that the optimum fiber orientation angle is 32^o with the meridional direction of the toroid. The results showed also that the optimum fiber orientation angle is 32^o with the meridional direction of the toroid.Finally, the study showed that carbon fiber/epoxy composites have the best performance among the other types of composite materials for sustaining high pressures with light weights.

I. INTRODUCTION

Pressure vessels are leak-proof containers used to store fluids (gases or liquids) under high pressures. These containers are used in many specific objects and engineering applications as: compressed natural gases containers, hydraulic cylinders, pneumatic vessels, space fuel tanks, hydrogen containers, and others. These vessels can have different configurations; cylindrical, spherical, elliptical, and toroidal shapes. Different materials are used for fabricating these vessels as: metallic (steel and aluminum), polymeric (polyethylene and PVC) and composite materials (fibrous materials). The selection of the material used to fabricate the pressure vessels depends on various factors as: the type of stored fluid, amount of the applied pressure and the effect of surrounding environmental conditions: Also, the design of these vessels is influenced by many other engineering parameters as the amount of applied internal pressure, geometry configuration and dimensions.

Many research works have been done to get the optimum design of that vessels. Azzam et al. [1] presented an optimum design for composite cylindrical pressure vessels having different fibrous layers bonded with polyester. In that paper, the optimum number of layers, the optimum fiber orientation of each layer and the required fiber tensioning in each layer during the winding process are determined for a specified applied internal pressure. Iliyas et al. [2] have studied the design and analysis of filament wound toroidal pressure vessels. The main objective of their study was to investigate and compare experimentally the deformations, stresses, buckling load multipliers, frequency values for conventional toroidal pressure vessels used to store CNG. They built a 3D-Numerical model (by Ansys) for toroidal pressure vessels wounded by advanced fibrous materials. They also performed theoretical calculations and analysis to determine stress and displacement values for toroidal pressure vessel.

Rakendu et al. [3] presented a finite element analysis for toroidal pressure vessels (using FEAST) for pressure vessels having different shapes of openings like manholes, hand holes, and nozzles. They found that due to that opening in the vessels' shells around the openings a stress concentration has raised because of geometrical discontinuity in the vessels. They tried to find the effect of diameter and position of openings on toroidal pressure vessels. They also studied the influence of the variation of stress concentration factor for different diameters of hole in order to find the effect of hole position.

Truong [4] has presented an optimization work to minimize the weight of toroidal pressure vessels using different evolution and particle swam optimization. In that paper, differential evolution and particle swarm optimization methods have been applied to the design of minimum weight toroidal shells subjected to internal pressures. The

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constraints of their optimization problem were first yield pressure, plastic pressures, plastic instability pressure and volume contained by the toroid. Their Optimality included geometry and wall thickness, which is constant or variable. Depending on their proposed geometry parameters of the toroid, the material saving could be as high as 72%.
Steele [5] studied weight and contained volume of toroidal pressure vessels with different types of cross section, such as circular, elliptic, modified elliptic, circumlunar, equal-stress and zero-hoop-stress cross-sections; the Tresca yield condition was used as the criterion in finding the minimum weight and the best ratio of contained volume to weight.
Bogomolnyi et at. [6] have formulated an analytical formula for wall-thickness of a toroid to minimize vessel's weight based on the condition of equal strength of shell without considering of the volume contained.
Marketos [7] has determined the optimal wrap angle associated with the geometry of the toroid meridian for filament wound toroidal pressure vessels.
In the invention of Mandel [8], a toroid was made of composite material with filament over-wrapped in order to optimize stresses in the toroid. The same objective was also given by Cook at al. [9], where the optimal design of a filament over-wrapped toroid could give 46% of the weight of a plain metal toroid for constant wall thickness based on elastic state in filamentary toroids.
Blachut [10] presented analytical, numerical, and experimental study to investigate the instability condition of toroidal shells. From his results, it is seen that there was no minimum weight design for the toroidal vessels based on the failure criterion of the plastic pressure and plastic instability pressure as constraints. In addition, the simultaneous use of the first yield pressure, plastic pressure, plastic instability pressure and the volume contained by vessel as constraints was not reported.
Purdel et al. [11] presented a stress analysis study for a toroidal shell used in flying platforms. The results of their study have been compared by a classical approach and by a numerical analysis and a good agreement is obtained.
Fowler et al. [12] presented a brief optimization review on toroidal pressure vessels. The review concluded that a small radii ratios ($1.25 < R/r < 3$) is required to maximize the potential space-saving and volumetric efficiencies of the torus.
In this paper, an optimum design of toroidal pressure vessels has been presented. The objective of study is to minimize the stresses induced in the vessel wall and so achieving the safety operation against failure. The parameters which optimized here are: the tube/toroid radii ratio, the tube thickness/toroid radius ratio and the orientation angle of the reinforcing fibers. Three different advanced fibrous materials (glass, Kevlar and carbon fibers) reinforcing epoxy have been manipulated leading to proposing the best type of reinforcing fibrous material.

II. STRESS ANALYSIS

2.1 Parameters Affecting Design of Toroidal Pressure Vessels

Figure (1) shows the main dimensions of a toroidal pressure vessel and the induced stresses in its wall due to the applied internal pressure (P_i). The tube wall will be subjected to three perpendicular stresses [2]: radial, meridional and hoop stresses, as shown in figure (1).

$$\sigma_r = -P_i \qquad (1)$$

$$\sigma_h = \frac{P_i r}{2t} \qquad (2)$$

$$\sigma_m = \frac{P_i r}{2t}\,\frac{2\mathrm{R}+\mathrm{r}\sin\emptyset}{\mathrm{R}+\mathrm{r}\sin\emptyset} \qquad (3)$$

Where;
s_r , s_h & s_m are radial, hoop and meridional stresses, respectively.
t, r & R are tube thickness, tube radius and toroid radius, respectively.

The above equations show that both the hoop and the radial stresses are fixed in all points around the circumference of the vessel, whereas the meridional stress is varying with the variation of the meridional angle f (see figure 1).

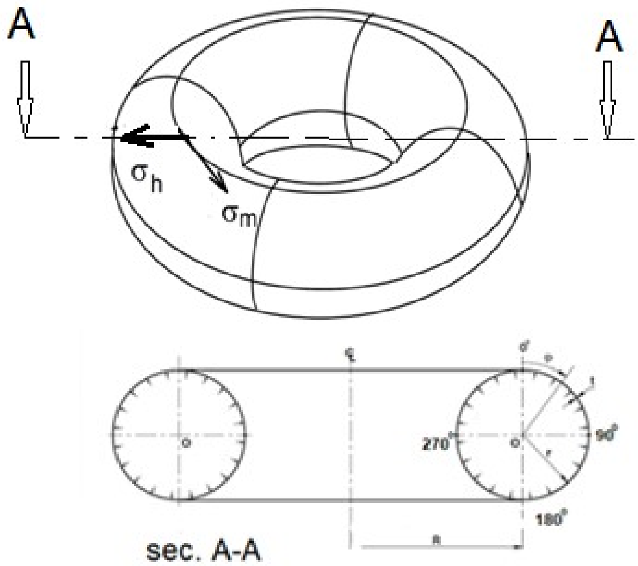

Figure (1) Toroidal vessel dimensions and stresses

2.2- Critical Zone of Maximum Stress

To find the critical position of the toroidal vessel, the maximum von-Mises stress has to be found where, the vessel starts to fail at the position of the maximum stress. Since both the hoop stress and radial stress are not changed across the circumference of the toroidal tube in all directions of the vessel (hoop or meridional directions) and therefore the position of critical zone will be at the maximum meridional stress.

To find the position of maximum stress, the meridional stress will be differentiated w.r.t. f as follows:

$$\frac{d\sigma_m}{d\phi} = 0 \tag{4}$$

$$\frac{d\sigma_m}{d\phi} = \frac{-rR\cos\phi}{(R+r)^2} = 0 \tag{5}$$

The above equation gives two solutions for f (90° & 270°), one to maximize the meridional stress and the other to minimize it. By checking $\frac{d^2\sigma_m}{d\phi^2} = +ve\ or - ve$ at both angles, one can realize that the maximum stress will be at f= 270° and the minimum stress will be at angle f= 90°. Therefore, the critical zone will be at the internal side of the toroid.

2.3- Optimum Dimensions of the Tube (radius and thickness)

The applied stresses acting on the toroidal vessels are in three perpendicular axes, i.e. all the stresses are principal stresses. The von-Mises stress will be as follows:

$$\sigma_{v.M} = \sqrt{\frac{(\sigma 1-\sigma 2)+(\sigma 2-\sigma 3)+(\sigma 3-\sigma 1)}{2}} \tag{6}$$

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Where;
s_1, s_2 & s_3 are the principal stresses
$s_1 = s_m$, $s_2 = s_h$ & $s_3 = s_r$

Since the value of s_3 is very small relative to the other two principal stresses, so it can be neglected. By substitution about $s_1 = s_m$ and $s_2 = s_h$, the von-Mises stresses at the high stress zone and at low stress zone could be obtained as follows:

1- von-Miss stress at the high stress zone (critical zone at f = 270°)

$$\sigma_{v.M}(max) = \frac{P_i r}{2t(R-r)}\sqrt{3R^2 - 3rR + r^2} \qquad (7)$$

2- von-Miss stress at the low stress zone (at f = 90°)

$$\sigma_{v.M}(min) = \frac{P_i r}{2t(R+r)}\sqrt{3R^2 + 3rR + r^2} \qquad (8)$$

To get the optimum values of the tube radius and thickness relative to toroid radius, the difference between the stresses in the maximum stress zone and minimum stress zone must be reduced. This ensures uniform stress distribution across the circumference of the vessel wall.
There are two ways to reduce the stress difference and material of toroids [4]. The first way is to keep the vessel wall thickness fixed and change the geometry of the toroid cross-section (R & r). The second way is to change both the wall thickness and the geometry of the cross-section. The first approach will be used in this study.

By putting r = R/r and g= t/r in von-Mises stress equations, equations (7) & (8) can be rewritten as follows:

$$\boldsymbol{\sigma_{v\cdot M}(max) = \frac{P_i}{2\gamma(\rho-1)}\sqrt{3\rho^2 - 3\rho + 1}} \qquad (9)$$

$$\boldsymbol{\sigma_{v\cdot M}(min) = \frac{P_i}{2\gamma(\rho+1)}\sqrt{3\rho^2 + 3\rho + 1}} \qquad (10)$$

Dividing Eqn. (9) by Eqn. (10), one can get the stress ratio (S_R) as follows:

$$\boldsymbol{S_R} = \frac{\sigma_{v,M}(\mathrm{Max})}{\sigma_{v,M}(\mathrm{Min})} = \boldsymbol{\frac{(\rho+1)}{(\rho-1)}\frac{\sqrt{3\rho^2-3\rho+1}}{\sqrt{3\rho^2+3\rho+1}}} \qquad (11)$$

The above equation shows that the change of the stress ratio, S_R, depends mainly on r.
To make a close uniform of stress distribution around the vessel wall, the stress ratio must be as minimum as possible.
To minimize the stress ratio, one can differentiate the above Eqn. (11) w.r.t. r and equate it by zero. The optimum value of **r** has been obtained as **2.6** (approximately) and the stress ratio at that value (**S_R**)**= 1.5** (approx.).
Hint: the second derivative has been checked at the optimum value of r and it was +ve, (which means minimum for S_R).

III. OPTIMUM FIBER ORIENTATION

Generally composite pressure vessels can sustain higher pressure loading with lighter weight than metallic ones. Therefore, most of aerospace pressure vessels, mobile tanks and under-water tanks are preferred to be made of composite materials. Different advanced fibrous materials as GRP, HS carbon/Epoxy and Kevlar/Epoxy are used for overwrapping pressure vessels [1]. The fibers are wounded around the toroidal vessel with different orientation angles relative to the meridional direction of the tube. To find the optimum angle of orientation, the applied stresses firstly have to be transformed to the principal directions of the layer (longitudinal and transvers) and secondly optimized to get the optimum values of fiber orientation angle.

3.1-Layer Stress Analysis

In the analysis of the toroidal pressure vessel, the overwrapped layers are assumed to have adjacent (±θ) angle lay-ups such that adjacent (±θ) lay-ups act as an orthotopic unit. The composite overwrapped layers forming the toroidal pressure vessel will be treated as a unidirectional laminated shell structure, i.e. all the layers have the same orientation angle of winding ±θ.

To transform the set of applied stresses (s_m & s_h) into the layer principal axes stresses (s_L & s_T), Fig. (2), one can use the following transformation matrix [1]:

$$\begin{pmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{pmatrix} = \begin{pmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{pmatrix} \begin{pmatrix} \sigma_m \\ \sigma_h \\ \tau_{mh} \end{pmatrix} \tag{12}$$

Where,

s_L, s_T = layer stresses in its principal axes

t_{LT} = layer shear stress

C = cos q & S= sin q , as shown in figure (2).

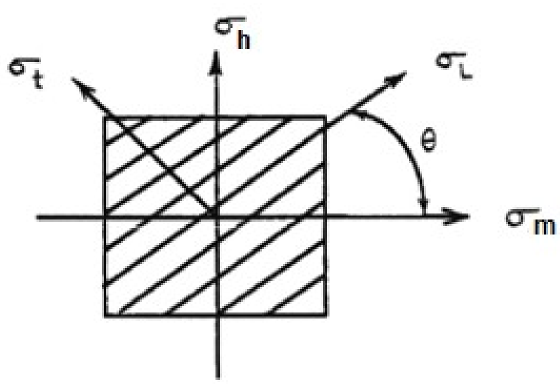

Figure (2) orthotropic lamina with its principal axes oriented by an angle q with reference axes

By substituting for σ_h, & σ_m from Eqn. (2) & Eqn. (3) into the above Eqn. (12), one can get the stresses σ_L, & s_T as follows:

$$\boldsymbol{\sigma_L = C^2\sigma_m + S^2\sigma_h = \frac{Pr}{2t}\left[\frac{2R-r}{R-r}\cos^2\theta + \sin^2\theta\right] \le S_L} \tag{13}$$

$$\boldsymbol{\sigma_T = S^2\sigma_m + C^2\sigma_h = \frac{Pr}{2t}\left[\frac{2R-r}{R-r}\sin^2\theta + \cos^2\theta\right] \le s_T} \tag{14}$$

$$S_L = v_f S_f + v_m S_m \tag{15}$$

$S_T = S_m$

Where;
S_L & S_T are the longitudinal and transverse ultimate strengths of the composite layer, respectively,
S_f & S_m are the fiber and matrix strengths, respectively.
v_f & v_m are the fiber and matrix densities, respectively.

According to above stresses, one can find more than one value for the failure pressure (P_i). The failure due to s_T is caused by the failure of the matrix itself or along the interface between the matrix and the fibers (initial failure) and results in the redistribution of stresses depending on the lamination angle. On the other hand, the failure due to σ_L is caused by the fiber fracture and results in complete ultimate failure (final failure).

So, in composite laminates the failure may be divided into two types: The first type is called first ply (initial) failure and is assumed to occur along the interface between the matrix and fiber or by failure of the matrix itself [1]. The second type of failure is called the ultimate (final) failure and is caused by the fracture due to fiber fracture. Referring to Fig.2, one can get the reasonable orientation angle of fibers [1] as given in Eqn. (16).

$$\tan^2\theta = \frac{\sigma_h}{\sigma_m} = \frac{R + r\sin\phi}{2R + r\sin\phi} \tag{16}$$

Since the optimum design has been obtained at R/r = 2.6 & t=0.1 r and at the critical zone of the tube (f= 270^o), the above equation gives the optimum angle of fiber orientation (q)= 32^o with meridional direction.

IV. CASE STUDY

The case study presented here shows the design of a toroidal pressure vessel based on the obtained optimum parameters. Three different advanced fibrous materials are alternatively used: E-glass, Kevlar-49 and HM-carbon fibers bonded by epoxy resin with 60% fiber volume fraction. The toroid dimensions are taken as R= 925 mm, r= 625 mm and the applied internal pressure P_i= 5.75 bars as the input data given in references [3 & 11]. The tube wall thickness, t, is assumed = 60 mm (about one-tens of tube radius). The mechanical properties of the used fibers and epoxy are given in table (1) [1].

Table (1) Typical properties of advanced fibers and epoxy resin [13]

Material	**Density** (kg/m^3)	**Ultimate strength** (MPa)
E-glass fibers	2500	1104
Carbon fibers (Thornel 300)	1800	1725
Kevlar-49 fibers	1400	1310
Epoxy resin	1200	100

V. RESULTS AND DISCUSSION

The Variation of von-Mises Stress with the meridional direction (f) has been presented in figure (3). It is shown that the critical zone of maximum stress is at f= 270^o whereas, the minimum stress is at f= 90^o. This explains why these types of pressure vessels starts to fail from the inner side of its toroid due to the high induced stresses. Figure (4) shows the effect of cross section of the toroid on the resulted stresses. It is sown that the stress ratio (S_R) decreases tremendously with increasing r (=R/r) such that at low values of r till 2.6 and then starts to stabilize at high values of it. This agrees well with the optimum value of r obtained in section 2.3. The optimum stress ratio at that value of r is 1.5.
Figure (5) shows the variation of principal stresses of composite overwrapped layers with the fiber orientation angle. It is shown that the longitudinal stress decreases with increasing the orientation angle whereas, the transverse stress increases with the orientation angle. This means that increasing the orientation angle of the fibers around the toroidal vessel with its meridional direction may tends to initialize the initial failure (due to matrix failure) of the vessel. On the other hand, decreasing the fiber orientation angle tends to initialize the ultimate failure of the vessel

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(due to fiber breakage). Therefore, using angle of 32^o as obtained from Eqn. (16) can enhance the design of the toroidal pressure vessel.

The variation of pressure/weight ratio with various ratios of r for different fibrous materials is shown in Fig. (6). It is shown that carbon/epoxy and Kevlar/epoxy composite layers have more higher ratios of pressure loading than glass/epoxy composite layers.

Although, carbon fibers have higher strength than Kevlar fibers, but there is a little difference between the carbon and Kevlar composite layers. This is due to that the density of Kevlar is less than of carbon fibers. Therefore, using carbon or Kevlar as overwrapped layers with epoxy matrix to reinforce toroidal vessels can give the same performance under pressure loading. On the other hand, using glass fibers gives less performance than carbon or Kevlar fibers.

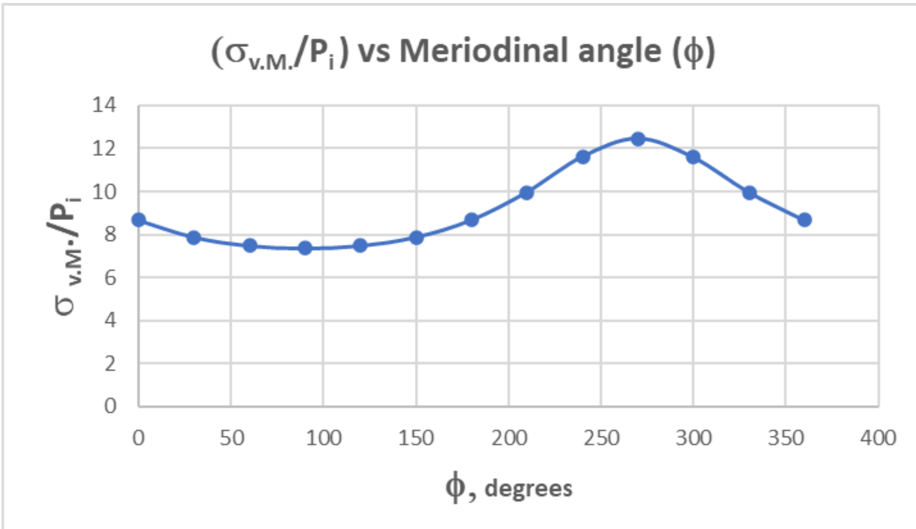

Figure (3) Variation of von-Mises Stress with the meridional direction

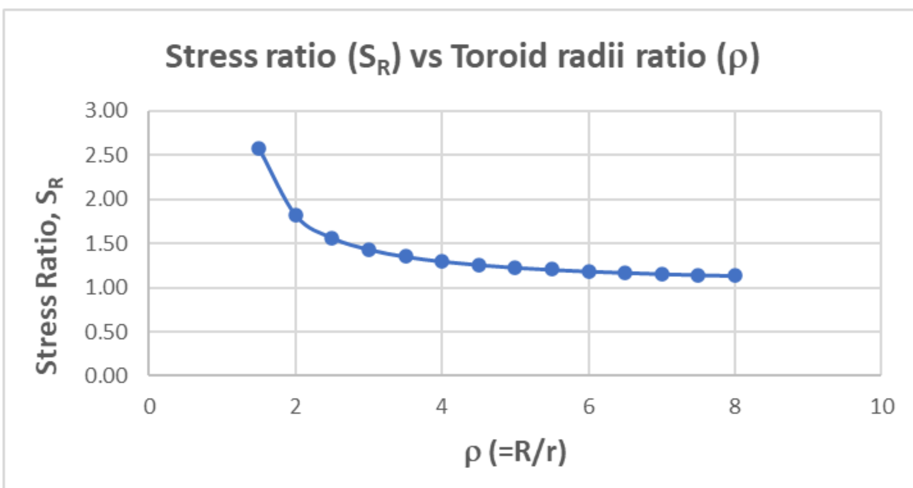

Figure (4) Variation of stress ratio (SR) with various ratios of r

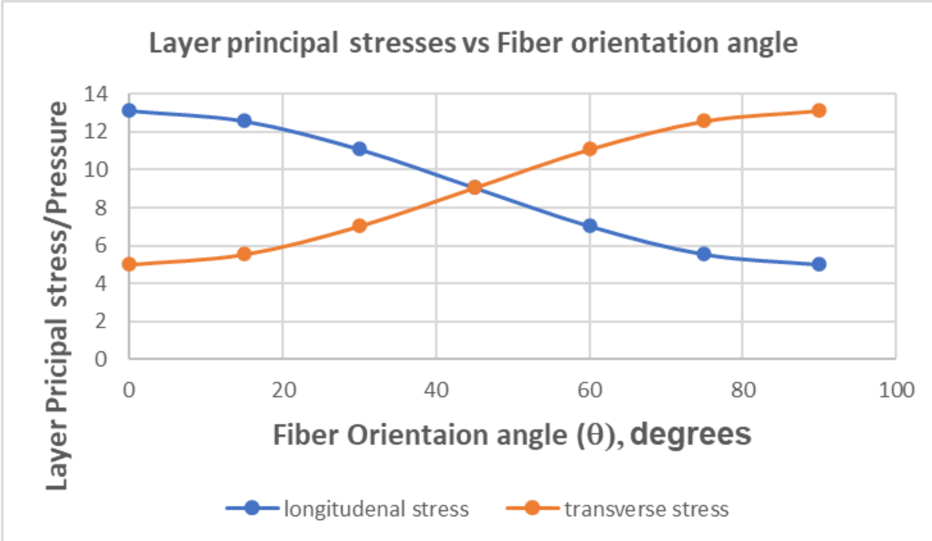

Figure (5) Variation of composite layer principal stresses with fiber orientation angle

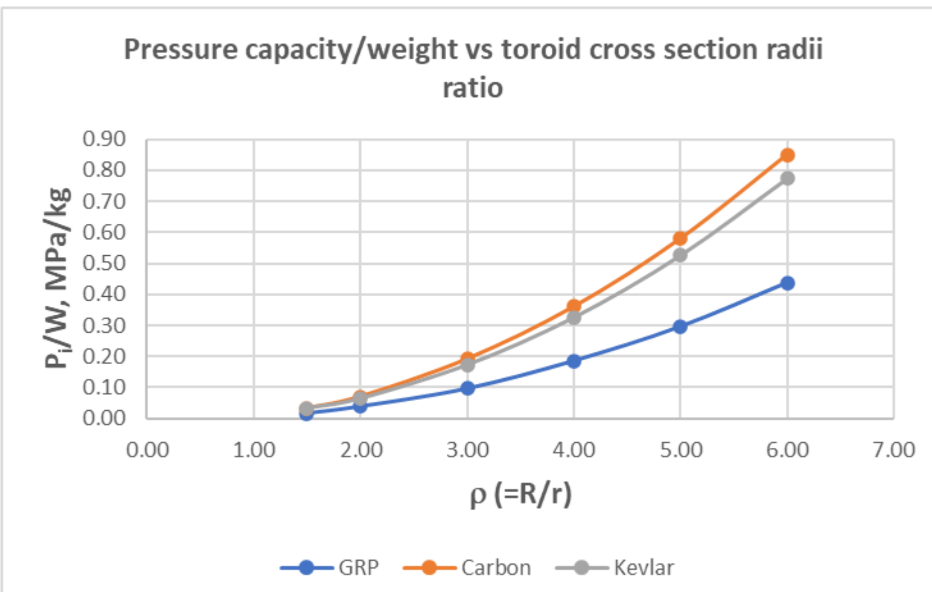

Figure (6) Variation of pressure/weight ratio with various ratios of r for different fibrous materials

VI. CONCLUSIONS

Throughout the analysis performed in this study, the following conclusions can be withdrawn:

1- The critical zone in the toroidal pressure vessels is at the inner side of the toroid, therefore it is recommended to avoid any stress concentration initiation at that zone
2- The optimum geometry of the vessel is at toroid/tube radii ratio of 2.6 (or more than that value).
3- The optimum orientation angle of the fibers around the vessel is 32^{o} with its meridional direction in order to decrease the longitudinal stress in fibers which may cause fibers breakage and so the ultimate failure of the vessel.
4- Using composite overwrapped layers around the vessels can enhance their performance more than the all-metallic vessels.
5- Using carbon fibers/epoxy or Kevlar fibers/epoxy composites in reinforcing toroidal pressure vessels helps to sustain high pressure loading with light weights of the toroidal pressure vessels.

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