Nonlinear dynamics of a harmonically excited axially moving beam

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Abstract - This research paper examines the complex transverse vibrations of axially moving beams subjected to harmonic excitation, which are vital elements in numerous industrial and structural applications. The study aims to comprehend the dynamic behavior and precisely determine frequency responses, which are crucial for enhancing design and performance. A groundbreaking method is employed to examine the transverse non-linear vibrations of a three-dimensional beam, incorporating mid-plane elongation effects to offer a thorough understanding of beam dynamics under non-linear conditions. The system's governing motion equations are discretized using Galerkin's method, yielding a set of interconnected non-linear differential equations, which are then analyzed using a multiple-time scale approach. The findings indicate that the three-dimensional vibrations exhibit bifurcation, dividing into two distinct non-linear branches. Furthermore, a hardening effect in the frequency response is observed due to the beam's middle plane stretching, resulting in a substantial increase in oscillation frequency. A parametric analysis identifies key factors influencing the onset of non-linear bifurcation, such as the damping coefficient and resonance deviation parameter, distinguishing between critical and safe bifurcation scenarios. A vibrometer was utilized to measure the frequency and other parameters. These results underscore the significant role of mid-plane stretching in amplifying non-linear effects, leading to higher oscillation frequencies and intricate vibration behaviors. This study provides valuable insights into axially moving beam dynamics, offering a foundation for improved design strategies and structural performance optimization in engineering applications. The outcomes emphasize the importance of considering non-linear phenomena in developing robust and efficient beam systems.

Keywords: Non-linear vibrations, axially moving beams, Harmonic excitation, Galerkin’s method, Three-dimensional vibrations, Vibrometer.

I. INTRODUCTION

Vibration equations can contain non-linear components originating from geometric, inertial, or material sources. Geometric non-linearity may arise from middle plane stretching and significant curvatures. A non-linear stress-strain relationship in the beam material leads to material non-linearity. Inertial non-linearity is caused by unevenly distributed and concentrated masses. Some researchers have employed multi-degree-of-freedom models or continuous media theory to simulate the vibration behavior of mechanical systems. Euler-Bernoulli's theory assumes that beam cross-sections stay perpendicular to the main axis during deformation, disregarding transverse vertical strains and shear deformations. Conducted a study examining the dynamic analysis of a unique three-degree-of-freedom mechanism consisting of two connected segments, subjected to external harmonic forces for motion induction. Their research thoroughly modeled the dynamical system's stability and vibration [1,2]. Utilized the homotropy analysis method to examine the non-linear behavior of a beam under axial load and develop an appropriate expression for the frequency of such nonlinear behavior.

The analysis of non-linear vibrations in beams plays a crucial role in structural engineering, facilitating precise evaluation and construction of beams subjected to dynamic forces. This is especially vital in industries such as aerospace, automotive, and civil engineering, where comprehending structural responses to various conditions is fundamental for guaranteeing safety and maximizing performance [1,2]. Beams, as essential structural elements, are extensively utilized across different scales. They are integral to cutting-edge materials and devices at micro and nano levels, while also being employed in large-scale structures like aircraft wings, satellite parts, and bridge spans [1,2]. Many structural components can be examined and modeled as beams. In the aerospace industry, beams are essential for structures such as airplane wings and satellite components, which must endure intricate dynamic forces [2,3].

Within the automotive field, vehicle parts, including the chassis, experience vibrations from road conditions and engine operations, necessitating sophisticated modeling approaches. Likewise, in civil engineering, beams constitute the core of major infrastructure, including bridges and building frameworks, where they withstand dynamic forces from traffic, wind, and earthquakes [2,3]. This modeling process aids in assessing a structure's stability and strength, serving as a preventive measure to avert accidents and failures. In physics and engineering, vibration denotes the oscillatory movement of an object around its equilibrium state. Linear vibration occurs when this movement follows a straightforward, predictable pattern directly proportional to the applied force. Conversely, nonlinear vibration happens when the relationship between the applied force and the resulting motion is not directly proportional. Nonlinear vibrations display characteristics such as multiple equilibrium points, jump phenomena, bifurcations, chaos, and harmonics, which are absent in linear systems. These vibrations are studied using sophisticated mathematical approaches, including perturbation techniques, numerical simulations, and phase plane analysis. The analysis of non-linear vibrations takes into account factors like material and geometric non-linearities, as well as boundary conditions [4]. This method is crucial for comprehending complex behaviors, such as resonance and stability, ensuring dependable performance across various applications. Non-linear terms in vibration equations can stem from geometric, inertial, or material sources. Geometric non-linearity arises from phenomena like middle plane stretching and large curvature occurrence. Material non-linearity is attributed to a non-linear stress-strain relationship in the beam's composition. Additionally, asymmetrically distributed or concentrated masses can result in inertial non-linearities. Some studies have examined the vibration behavior of mechanical systems using models with multiple degrees of freedom or by applying continuous media theory [7-10]. Euler-Bernoulli theory posits that a beam's cross-sections remain perpendicular to its main axis during deformation [13], disregarding transverse vertical strains and shear deformations. Research by investigates the dynamic behavior of an innovative three-degree-of-freedom mechanism comprising two interconnected segments [14]. The system is actuated by external harmonic forces to generate motion. performed a comprehensive modeling of the system's stability and vibration dynamics. Utilized the homotropy analysis method to examine the non-linear behavior of a beam under axial loading and formulated an appropriate expression to represent the frequency associated with this non-linear behavior [16]. The nonlinear dynamics of axially loaded systems explores how structures or mechanical components behave under axial forces when nonlinear effects become significant. Axial loading can induce instabilities, substantial deformations, and intricate vibrational behavior, resulting in nonlinear dynamic responses. Nonlinear dynamics can occur when the stress-strain relationship is not linear, meaning the material does not follow Hooke's Law. Large deformations can alter the system's geometry, affecting its stiffness and stability. Time-varying axial forces can act as parametric excitations, leading to resonances, subharmonic responses, and instability phenomena. For instance, cables under axial tension exhibit nonlinear dynamic behaviors, including large oscillations and snap-through motion.

II. RESEARCH OBJECTIVE

The primary objective of this research is to investigate the complex transverse vibrations of axially moving beams subjected to harmonic excitation, which play a crucial role in various industrial and structural applications. This study aims to develop a comprehensive understanding of the dynamic behavior of such beams, with a particular focus on accurately determining their frequency responses. By incorporating mid-plane elongation effects, the research seeks to analyze the influence of non-linearities on beam vibrations, providing deeper insights into their dynamic characteristics. A key goal is to discretize the governing equations of motion using Galerkin's method, leading to a set of coupled non-linear differential equations that are subsequently analyzed using a multiple-time scale approach. Through this methodology, the study aims to identify bifurcation phenomena, characterize frequency response behaviors, and examine the effects of parameters such as the damping coefficient and resonance deviation on non-linear bifurcation. Additionally, the research strives to quantify the hardening effect observed in the frequency response due to mid-plane stretching, which significantly impacts oscillation frequencies. By conducting parametric analyses and utilizing experimental validation with a vibrometer, this study intends to enhance the understanding of non-linear vibration behaviors and provide critical insights for optimizing beam design and improving structural performance in engineering applications. The findings aim to contribute to the development of robust, efficient, and resilient beam systems by emphasizing the importance of considering non-linear dynamics in their design and analysis.

III. THEORETICAL ASPECTS

Key Aspects of Nonlinear Dynamics in Axially Moving Beams

Axial Motion: Axially moving beams are structures that experience motion along their longitudinal axis, such as conveyor belts, saw blades, paper webs, or cable-driven systems. The axial velocity introduces gyroscopic and inertial effects, making the governing equations more complex. Fig 1. Resemble an axially moving beam for transverse displacement of Euler- Bernoulli beam under parametric excitation.

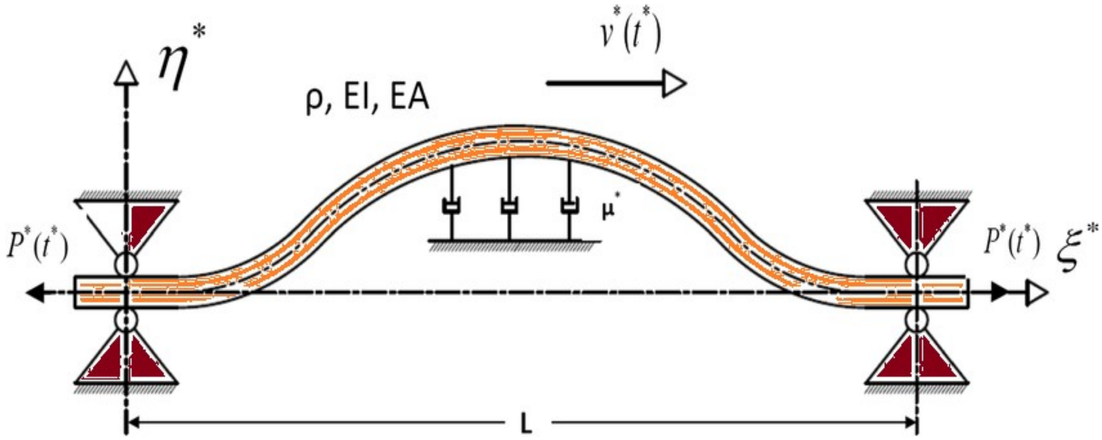


Fig 1. Schematic diagram of an axially moving beam (source: Internet)

The transverse displacement w(x,t) of the Euler-Bernoulli beam under parametric excitation is governed by the following equation:
ρA times the second partial derivative of w with respect to t, plus c times the first partial derivative of w with respect to t, plus EI times the fourth partial derivative of w with respect to x, plus T times the second partial derivative of w with respect to x, is equal to f(x,t).
Mathematically, this can be written as:

$$\rho A \frac{\partial w^2}{\partial t^2} + c\frac{\partial w}{\partial t} + EI\frac{\partial w^4}{\partial x^4} + T\frac{\partial w^2}{\partial x^2} = f(x,t)$$

Where:
ρ is the mass density of the beam,
A is the cross-sectional area,
c is the damping coefficient,
E is Young’s modulus,
I is the moment of inertia,
T is the axial tension,
f(x,t) represents the external force or excitation.

Harmonic Excitation: The beam is subjected to time-dependent periodic forces (e.g., sinusoidal forces) that cause transverse vibrations. These forces can lead to resonances, amplifying the vibrations under certain conditions.

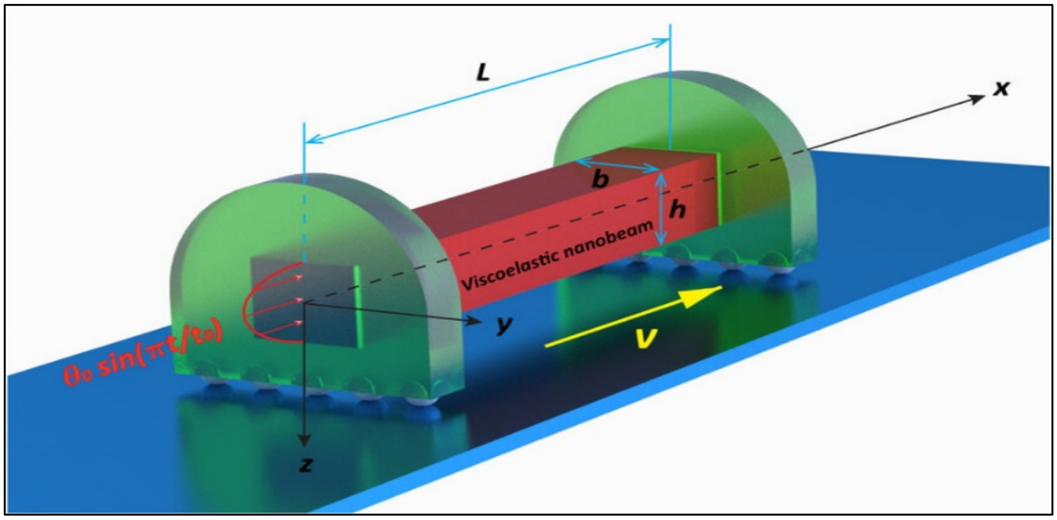


Fig 2. Viscosity nanobeam (source: Internet)

Harmonic excitation in a beam is a fundamental concept in structural dynamics, affecting stability, safety, and performance. Understanding its effects, especially resonance and damping, is crucial for designing structures that can withstand periodic loads without excessive vibrations as shown in Fig 2.
A harmonically excited beam is subjected to a periodic force or displacement of the form:

$$f(x,t) = F_0\cos(\omega t + \phi)$$

where,
F_0 is the amplitude of the applied force,
ω is the excitation frequency (in radians per second),
ϕ is the phase angle,
t is time.
Alternatively, harmonic excitation can be in the form of boundary conditions, such as a sinusoidal base motion:

$$w(0,t) = A\cos(\omega t)$$

where, A is the amplitude of the applied displacement.
Nonlinear Effects:
Geometric Nonlinearity: Due to large deformations or mid-plane stretching, where the relationship between strain and displacement becomes nonlinear.
Inertial Nonlinearity: Arises due to the effect of distributed or concentrated masses.
Material Nonlinearity: Occurs if the stress-strain relationship of the beam material is non-linear.
Nonlinear effects caused by mid-plane stretching influence the axial tension TTT, modifying it as follows:

$$T_{nonlinear} = \frac{EA}{2L}\int_0^L (\frac{\partial w}{\partial x})^2 dx$$

Where, L denotes the beam's length.
This nonlinear term contributes to geometric stiffness, affecting the beam's vibration dynamics.
Boundary Condition Nonlinearity: The boundary conditions for a beam depend on its physical configuration, such as clamped-clamped or simply supported conditions. For simply supported ends, the boundary conditions are:

$$w(0,t) = w(L,t) = 0$$

$$\frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0$$

Internal Resonance: In nonlinear systems, energy can transfer between different vibration modes (e.g., three-to-one internal resonance), significantly affecting the beam's response.
To facilitate analysis, the equation is commonly transformed into a dimensionless form using the following non-dimensional variables:

$$\bar{x} = \frac{x}{L}, \quad \bar{t} = \frac{t}{\sqrt{\frac{\rho A L^4}{EI}}}, \quad \bar{w} = \frac{w}{L}$$

Dynamic Stability and Bifurcation: Nonlinearities can cause dynamic instability, leading to phenomena like bifurcations (qualitative changes in system behavior), chaotic vibrations, or large amplitude oscillations as shown in Fig 3.

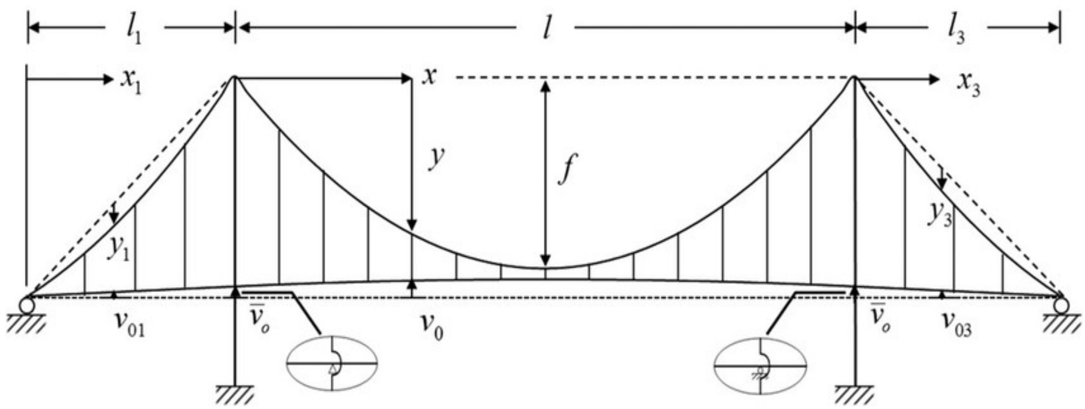


Fig 3. Mechanism of suspension bridge (source: Internet)

Using the multiple-time scale method, the solution is expressed as:

$$w(x,t) = w_0(x, t_0, t_1) + \epsilon w_1(x, t_0, t_1) + \epsilon^2 w_2(x, t_0, t_1) + \cdots$$

where t_0= t represents the fast time scale, and t_1= ϵt corresponds to the slow time scale.
This method consists of the following steps:
Substituting the expansion into the governing equation, separating terms based on different orders of ϵ, iteratively solving the resulting linear and nonlinear equations.

IV. METHODOLOGY

The governing equations for such systems are typically derived using: Euler-Bernoulli Beam Theory: Assumes the beam's cross-sections remain perpendicular to the neutral axis but may not fully capture nonlinear behaviors. Timoshenko Beam Theory: Includes shear deformation and rotational effects for more accurate modeling. The equation of motion is typically a partial differential equation (PDE) that incorporates: Axial motion terms, External harmonic excitation forces, Nonlinear terms resulting from geometric, material, or inertial factors.
The Galerkin Method is a numerical technique widely used in mechanical vibrations to approximate the solution of differential equations, particularly those arising in structural and dynamic systems. It is a projection method that transforms partial differential equations (PDEs) governing the motion of a system into a set of ordinary differential equations (ODEs). This method is particularly effective in analyzing systems with complex geometries or boundary conditions, such as beams, plates, and shells.

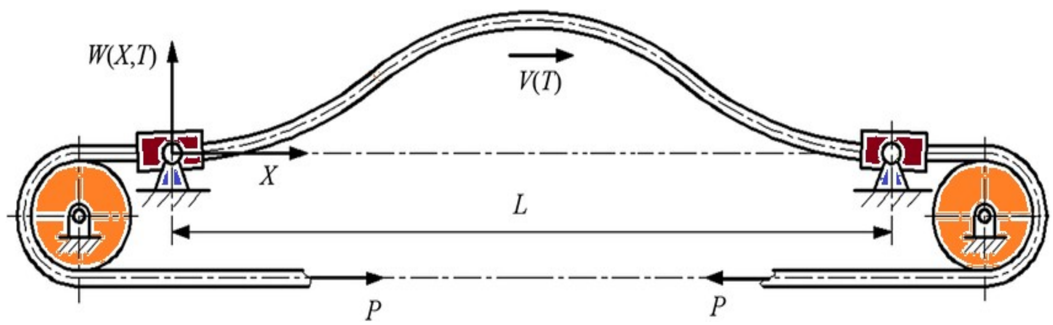


Fig 4. Skeletal mechanism of conveyer belt (source: Internet)

The geometric model of the clamped-clamped beam and coordinate system are shown in Fig. 5. Partial differential equations controlling the Euler-Bernoulli beam's transverse vibrations, taking into account the variable axial force along the length of the beam, are obtained as follows [17]

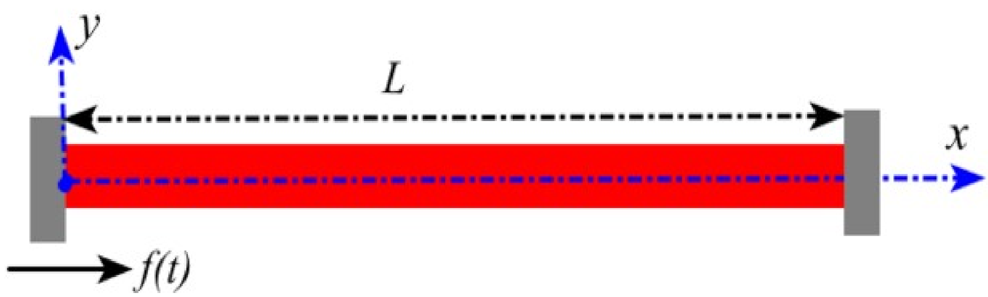


Fig 5. Geometric of the clamped-clamped Euler-Bernoulli beam [2]

$$\frac{\partial T(x,t)}{\partial x} = px$$

$$m\frac{\partial^2 V(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left[\frac{T(x,t)\ \partial V(x,t)}{\partial x}\right] + \frac{\partial^2}{\partial x^2}\left[\frac{EI\ \partial^2 V(x,t)}{\partial x^2}\right] = py \quad (1)$$

$$m\frac{\partial^2 W(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left[\frac{T(x,t)\ \partial W(x,t)}{\partial x}\right] + \frac{\partial^2}{\partial x^2}\left[\frac{EI\ \partial^2 W(x,t)}{\partial x^2}\right] = pz$$

where,
V(x, t) and W(x, t): Simplified notations for transverse displacements in the y and z directions, respectively.
ρ: Mass density per unit length (ρ = m)
q_1 and q_2: Distributed external forces per unit length in the y and z directions, respectively (q_1=py, q_2=pz).
T (x, t): Tension in the beam at position x and time t.
E: Young's modulus of the beam material.
I: Moment of inertia of the beam cross-section.
m: Mass per unit length of the beam.
p: Distributed external force acting on the beam in the y and z directions.

To transform the partial differential equation (Eq. 1) into a system of ordinary differential equations, the Galerkin method is employed. This involves assuming a solution form for Eq. (1),

$$V(x,t) = \Phi_1(x)v(t), W(x,t) = \Phi_2(x)w(t) \quad (2)$$

To apply the Galerkin method to the current problem, the spatial functions $\Phi_1(x)$ and $\Phi_2(x)$ in Eq. (2) are chosen to represent the linear vibration modes of the beam in the transverse directions. Consequently, v(t) and w(t) represent the time-varying amplitudes of these beam vibrations. The boundary conditions for the beam depicted in Fig. 1 are as follows.

$$V(0,t) = \frac{\partial V(0,t)}{\partial x} = 0, \quad V(L,t) = \frac{\partial V(L,t)}{\partial x} = 0 \quad (2a)$$

$$W(0,t) = \frac{\partial W(0,t)}{\partial x} = 0, \quad W(L,t) = \frac{\partial W(L,t)}{\partial x} = 0 \quad (2b)$$

$$U(0,t) = 0, U(L,t) = s_0 + f(t) \quad (2c)$$

In the given equations, s_0 and f(t) represent the initial static displacement and the dynamic displacement applied to the left support of the beam, respectively. Based on the defined boundary conditions, the upper support of the beam initially experiences a static displacement. Then, during vibrations, it undergoes excitation in the form of a dynamic displacement through the upper support. From Equation (2a), an expression for the axial force can be derived. By applying mathematical operations and assuming that variations along the beam's length, volume forces, and other external forces are negligible, the axial force can be determined as follows:

$$T(t) = \frac{EAU_0}{L} + \left(\frac{EA}{L}\right) * \left[f(t) + \left(\frac{1}{2}\right)\int_0^L \left(\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right) dx\right] \quad (3)$$

As can be seen in Eq. (3), the obtained axial force is only a function of time. In the continuation of using the Galerkin method, it is assumed that [18,19]:

$$\varphi_1(x) = \varphi_2(x) = \cosh(\beta x) - \cos(\beta x) + \frac{(\cosh(\beta L) - \cos(\beta L))}{(\sinh(\beta L) - \sin(\beta L))}[\sinh(\beta x) - \sin(\beta x)] \quad (4)$$

where:
$\varphi_1(x)$ and $\varphi_2(x)$: Spatial functions representing the mode shapes of the beam vibrations in the y and z directions, respectively.
β: A constant related to the natural frequency of the beam.
x: Position along the length of the beam.
L: Length of the beam.

By inserting Eq. (3) into Eq. (1), the following equation can be written:

$$EI\frac{\partial^4 V}{\partial x^4} - \left(\frac{EA}{L}\right) * \frac{\partial}{\partial x}\left[\left(s_0 + f(t) + \left(\frac{1}{2}\right)\int_0^L \left(\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right) dx\right) * \frac{\partial V}{\partial x}\right] + m\frac{\partial^2 V}{\partial t^2} = 0 \quad (5a)$$

$$EI\frac{\partial^4 W}{\partial x^4} - \left(\frac{EA}{L}\right) * \frac{\partial}{\partial x}\left[\left(s_0 + p(t) + \left(\frac{1}{2}\right)\int_0^L \left(\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right) dx\right) * \frac{\partial W}{\partial x}\right] + m\frac{\partial^2 W}{\partial t^2} = 0 \quad (5b)$$

Utilizing Equation (4), the boundary conditions for the beam are established. Subsequently, by applying the Galerkin method to the derived equations, a system of ordinary differential equations is obtained. In the implementation of the Galerkin method, the weight function is selected to be identical to the first linear mode shape of a clamped-clamped beam.

$$\ddot{v}(t) + (\omega_0^2 + \phi f(t))v(t) + \beta(v^3(t) + v(t)w^2(t)) = 0$$

$$\ddot{w}(t) + (\omega_0^2 + \phi f(t))w(t) + \beta(w^3(t) + w(t)v^2(t)) = 0 \quad (6)$$

Equation (6) represents the characteristic equation governing the beam's vibrations. The coefficients within this equation are detailed in the appendix. The subsequent section will delve into the solution methodology for this system of equations. The analysis will be conducted under the assumption of zero initial conditions.

V. RESULT AND DEVELOPMENT

A. Displacement of beam bifurcation graph generated using PYTHON language:
Shown in fig 6. This graph showed variation of vibration over single degree of freedom and provided a pattern to analyze the displacement of a beam.

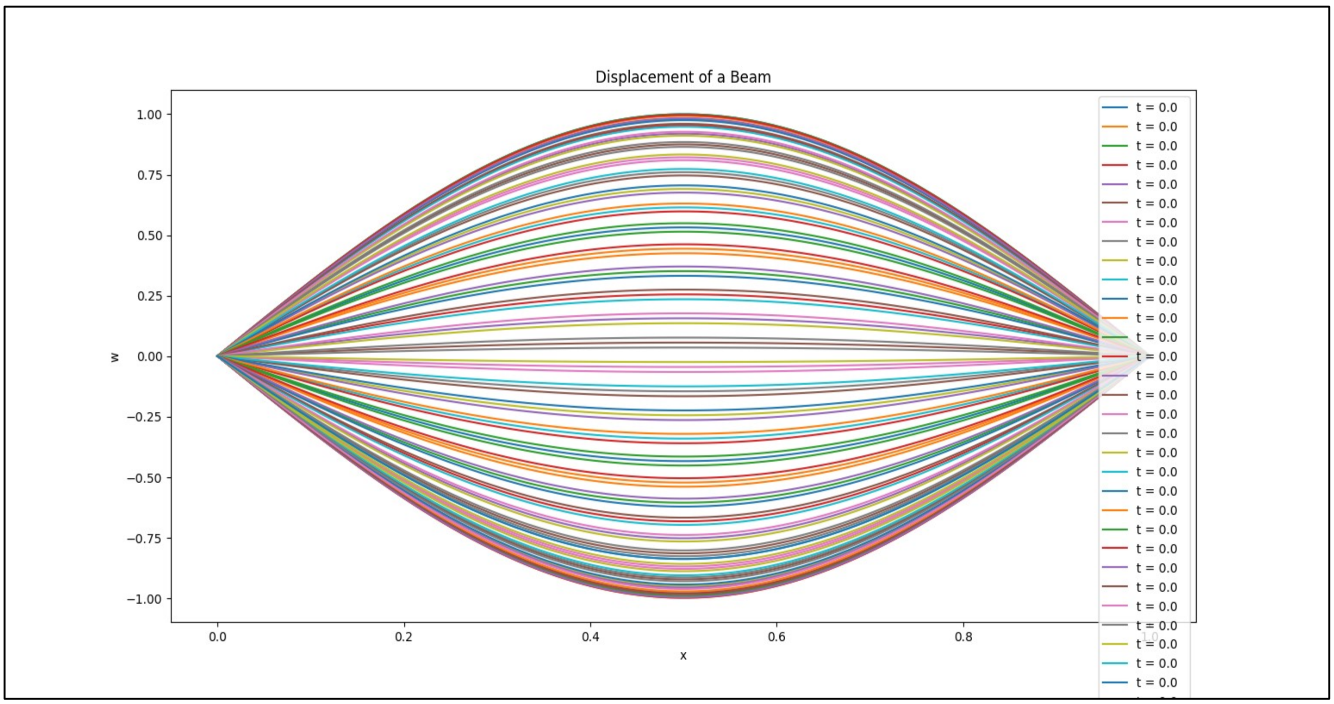


Fig 6. Displacement of beam

B. Oscillation of the system found using vibrometer device and PYTHON language:
Generated the image using VIBROMETER and compiled in PYTHON language for final output. This shows
Plot (a): The time series could represent the oscillation of a physical system like a pendulum or a mass-spring system. The periodic fluctuations suggest a stable oscillation.
Plot (b): The phase portrait could be a representation of the same system as in plot (a). The closed loops in the phase portrait indicate that the system's trajectory in the phase space is periodic, confirming the oscillatory behavior observed in the time series.

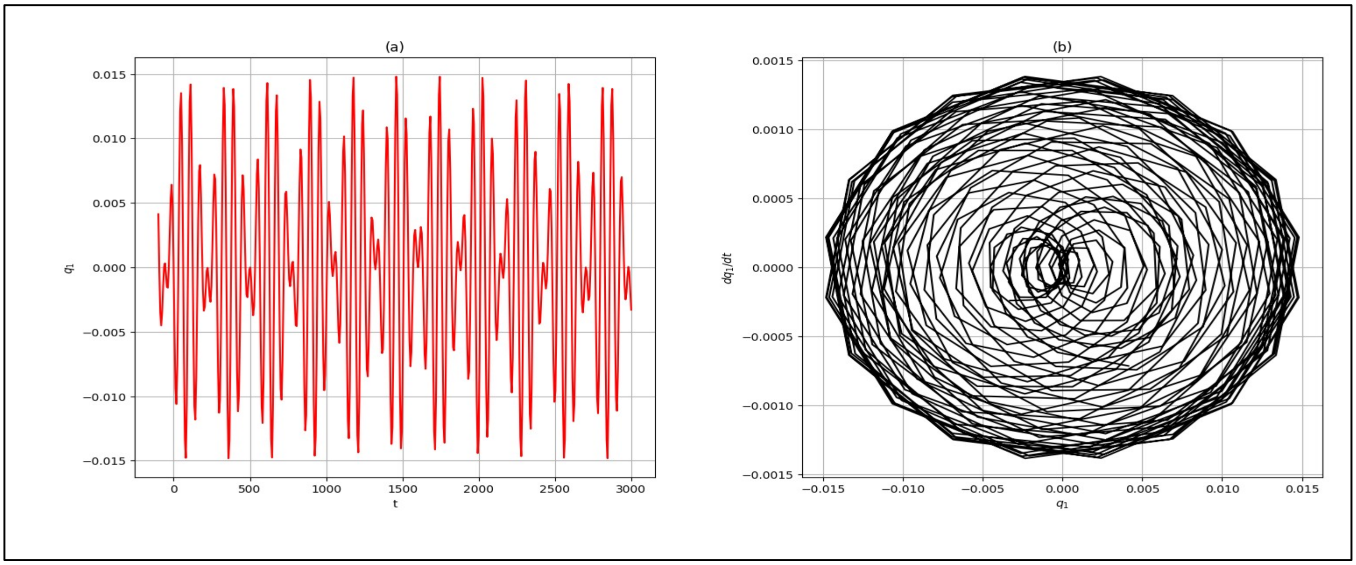


Fig 7. Oscillation graph

C. Developed a VIBROMETER in low cost for vibration analysis:
It calculates displacement of vibration according to time in X and Y plane.
It is portable to use and can be connected to PC or laptop for easy access of data.

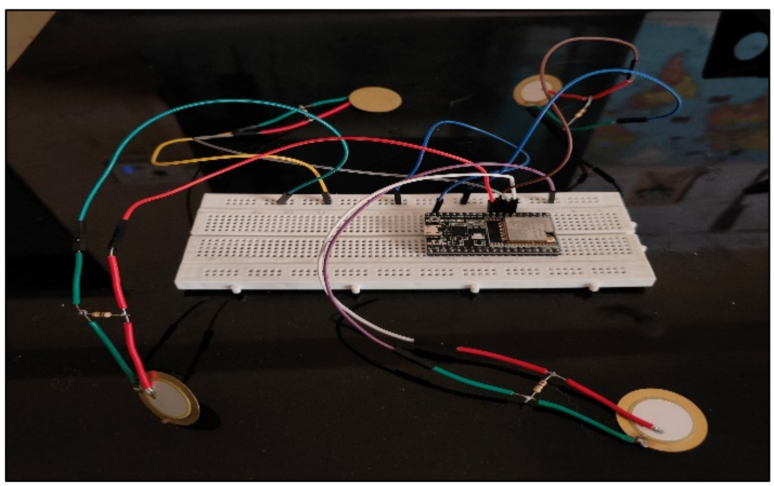


Fig 8. Vibrometer

VI. 3D SOLID WORK MODELLING

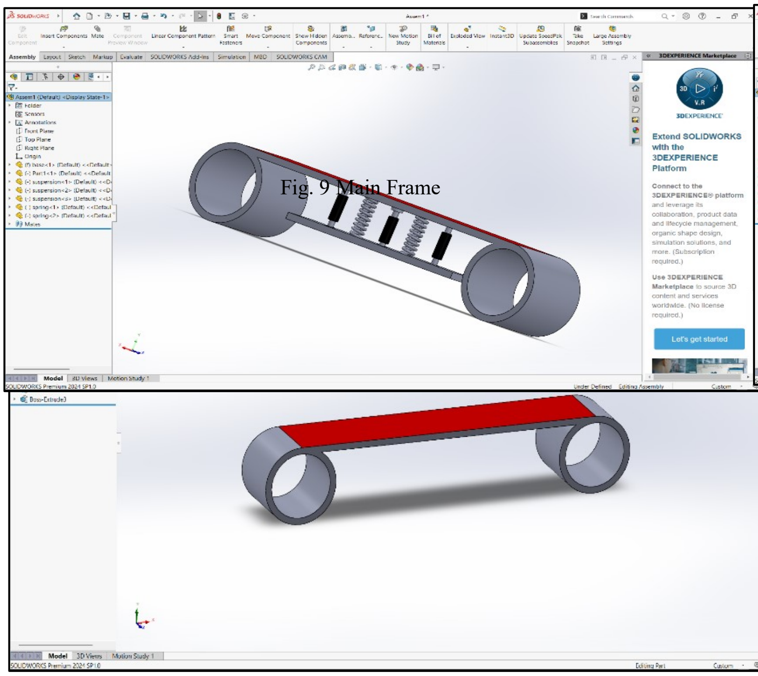


Fig. 9 Main Frame

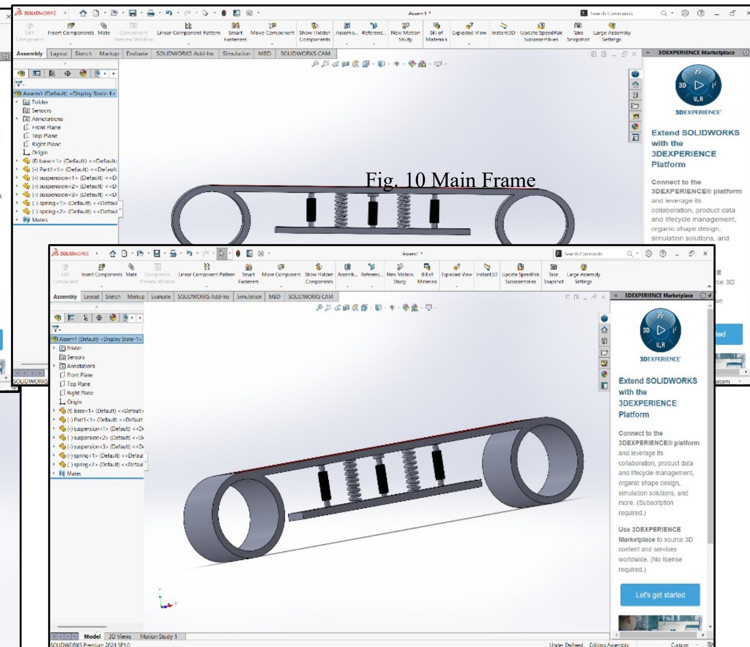
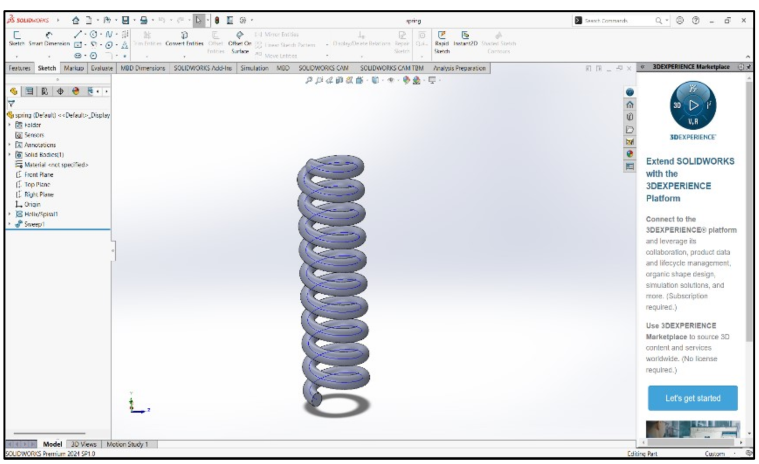


Fig. 10 Main Frame



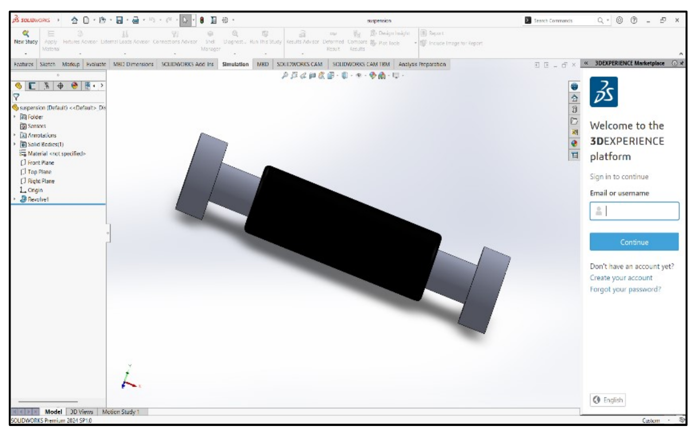


Fig. 13 Spring Coil

Fig. 14 Shock absorber

VII. SUMMARY

This study analyzes the non-linear transverse vibrations of beams, which are critical for designing and optimizing industrial and construction structures. The research introduces a novel approach by incorporating the effects of mid-plane elongation into the transverse non-linear vibration analysis of a three-dimensional beam. Using a multiple-time scale method, the study provides insights into the dynamic behavior of beams under non-linear influences.

Key findings reveal the phenomenon of frequency response splitting into two non-linear branches during three-dimensional vibrations, along with a hardening behavior in the frequency response caused by mid-plane stretching. A parametric study highlights the impact of the damping coefficient and resonance deviation parameter on the onset of critical versus safe bifurcations. Additionally, mid-plane stretching increases the non-linear term coefficient in the vibration equations, raising the beam's oscillation frequency. These findings contribute to a deeper understanding of beam dynamics under non-linear effects.

VIII. CONCLUSION

Summary of Key Findings: The study systematically analyzed the nonlinear dynamics of an axially moving beam under harmonic excitation. Both sub-critical and super-critical behaviors were identified, revealing distinct dynamic characteristics in each regime.
Comparison with Existing Models: Results were consistent with theoretical predictions and extended prior work by incorporating complex nonlinear effects. The study demonstrated enhanced accuracy through improved modeling techniques and computational simulations.
Practical Implications: Findings provide valuable insights for the design and operation of axially moving systems in engineering, such as conveyor belts, saw blades, and cables. Recommendations for parameter optimization to avoid undesirable dynamics were proposed.

Future Research Opportunities: Investigating higher-dimensional systems and incorporating advanced boundary conditions could broaden the applicability of the findings. Experimental studies are recommended to validate theoretical predictions and refine the model for real-life engineering scenarios.
Contributions to the Field: This research provides a comprehensive understanding of the nonlinear and bifurcation dynamics of Euler-Bernoulli beams under parametric excitation, contributing significantly to the field of structural dynamics.

IX. FUTURE WORK

Experimental Validation: Conduct experimental studies to validate the theoretical and numerical predictions presented in this study. Develop prototypes of axially moving beams with controlled harmonic excitation for detailed testing. Design and develop a wireless VIBROMETER for vibrational analysis
Multi-Mode Analysis: Expand the analysis to consider interactions between multiple vibration modes, particularly in higher-frequency regimes.
Numerical Efficiency Improvements: Develop more computationally efficient algorithms for simulating nonlinear dynamics in large-scale systems. Leverage machine learning techniques for faster prediction and analysis of bifurcation scenarios.

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